Blacktown Boys' High School

2020

HSC Trial Examination

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 7)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 14)
- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ____

Teacher Name: ____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2020 Higher School Certificate Examination.

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1 Which of the following is equivalent to i^{2020} ?

A. i^{100}

B. $(i^{11})^5$

C. $(i^{21})^5$

D. i^{10}

Q2 Given that $z = 5e^{\frac{i\pi}{6}}$, which expression is equal to $(\bar{z})^{-1}$?

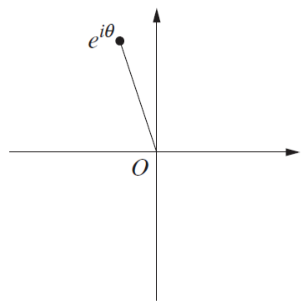
A. $\frac{1}{5}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

B. $5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

C. $\frac{1}{5}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

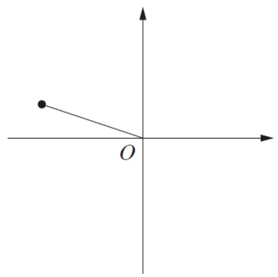
D. $5\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

Q3 The Argand diagram shows the complex number $e^{i\theta}$.

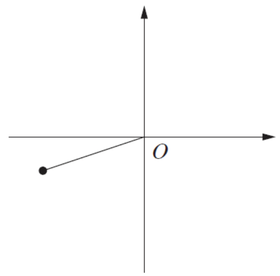


Which of the following diagrams best shows the complex number $ie^{-i\theta}$?

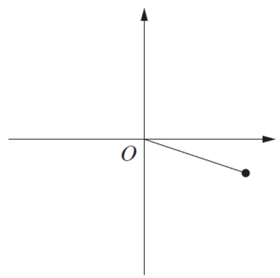
A.



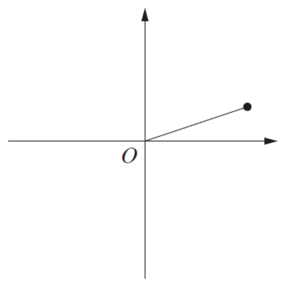
B.



C.



D.



Q4 Which of the following statement is true?

A. $\forall x \in R, \exists y \in R$ such that $xy = 10$

B. $\forall x \in R, \exists y \in R$ such that $y = x^2$

C. $\exists x \in R$, such that $\forall y \in R$, $xy = 10$

D. $\exists x \in R$, such that $\forall y \in R$, $y = x^2$

Q5 The following slogan is used at a particular swimming centre.

"If you are a strong swimmer, then you are unlikely to drown."

Which of the following statements is logically equivalent to the above slogan?

A. If you are unlikely to drown, then you are a strong swimmer.

B. If you are a strong swimmer, then you are likely to drown.

C. If you are not a strong swimmer, then you are likely to drown.

D. If you are likely to drown, then you are not a strong swimmer.

Q6 Using a suitable substitution $\int_a^b x^9 e^{3x^{10}} dx$, where a and b are real constants, can be written as

A. $\int_a^b e^{3u} du$

B. $\frac{1}{10}\int_{a^{10}}^{b^{10}} e^{3u} du$

C. $\frac{1}{30}\int_{30a^9}^{30b^9} e^{u} du$

D. $\frac{1}{3}\int_{3a^{10}}^{3b^{10}} e^{u} du$

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Q7 Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + 10\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}$, the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ is

A. $-\frac{13}{\sqrt{43}}\left(2\underset{\sim}{i} - 5\underset{\sim}{j} + 10\underset{\sim}{k}\right)$

B. $-\frac{13}{43}\left(2\underset{\sim}{i} - 5\underset{\sim}{j} + 10\underset{\sim}{k}\right)$

C. $-\frac{39}{\sqrt{35}}\left(3\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}\right)$

D. $-\frac{39}{35}\left(3\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}\right)$

Q8 A particle undergoing simple harmonic motion in a straight line has an acceleration of $\ddot{x} = 100 - 25x$, where x is the displacement after t seconds.

Where is the centre of motion?

A. $x = 10$

B. $x = 5$

C. $x = 4$

D. $x = 2$

Q9 A particle is moving in a straight line with velocity at any particular time given by

$$v = 2\tan^{-1} x$$

Which of the following represents the acceleration of the particle?

A. $\frac{2}{\sqrt{1 - x^2}}$

B. $\frac{4}{1 + x^2}$

C. $\frac{4\tan^{-1} x}{1 + x^2}$

D. $-2\operatorname{cosec}^2 x$

Q10 Let $\underset{\sim}{r}(t) = \left(1 - \sqrt{a}\cos t\right)\underset{\sim}{i} + \left(1 - \frac{1}{b}\sin t\right)\underset{\sim}{j}$ for $t \geq 0$ and $a, b \in R^+$ be the path of a particle moving in the Cartesian plane.

The path of the particle will always be a circle if

A. $a^2b = 1$

B. $ab^2 = 1$

C. $ab = 1$

D. $a + b = 1$

End of Section I

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Let $z_1 = 1 - 2i$ and $z_2 = 3 + 4i$. Express the following in the form $a + bi$ where a and b are real numbers.

i) $z_1 - z_2$ **1**

ii) $z_1 \times \overline{z_2}$ **1**

iii) Show that $\dfrac{2+i}{z_2} - \dfrac{z_1}{5i}$ is a real number. **2**

b) Let $z = x + yi$, where x and y are real numbers, and $2z + \bar{z} = 21 - 8i$. Find the values of x and y. **2**

c) i) Write $-\sqrt{3} + 3i$ in modulus argument form. **2**

ii) Hence express $\left(-\sqrt{3} + 3i\right)^5$ in the form $a + bi$, where a and b are real numbers. **2**

d) i) Solve for x and y, where x and y are real numbers, given that **2**

$$(x + iy)^2 = 9 - 40i$$

ii) Hence or otherwise, solve $z^2 - 7z + 10 + 10i = 0$. **3**

End of Questions 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find

i) $\displaystyle\int \frac{2dx}{x^2 - 8x + 21}$ **2**

ii) $\displaystyle\int x^2 \log_e x \, dx$ **2**

b) Use integration to show that **3**

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx = \log_e \sqrt{e^{4x} + 1} - x + c$$

c) i) Find the values of A, B and C such that **2**

$$\frac{6x^2 + 7x - 3}{(x-5)(x^2+1)} = \frac{A}{x-5} + \frac{Bx + C}{x^2 + 1}$$

ii) Hence find $\displaystyle\int \frac{6x^2 + 7x - 3}{(x-5)(x^2+1)} dx$ **2**

d) i) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$ **1**

ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin 9x \cos 6x \, dx$ **3**

End of Questions 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) The coordinates of three points are $A(4, -2, -3)$, $B(0, -1, 4)$ and $C(-4, 4, 1)$.

i) Find $\overrightarrow{AB}$. **1**

ii) The points A, B and C are the vertices of a triangle. Prove that this triangle has a right angle at B. **2**

iii) Find the exact length of the hypotenuse of this triangle. **1**

b) i) Show that the points $L(-5, 6, -5)$, $M(7, -2, -1)$ and $N(10, -4, 0)$ are collinear. **2**

ii) Hence find the ratio in which M divides LN. **1**

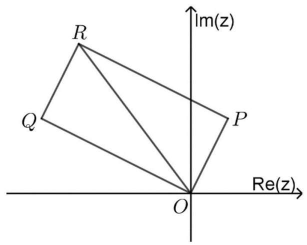
c) Sketch the region in the complex plane showing the inequalities **3**

$1 \leq |z - 1 + i| \leq 2$ and $\dfrac{3\pi}{4} \leq \arg(z - 1 + i) \leq \pi$

Question 13 continues on next page

Question 13 (continued)

d) Let $z = \cos\alpha + i\sin\alpha$, where α is an angle in the first quadrant. On the Argand diagram the point P represents z, the point Q represents $i\sqrt{3}z$ and the point R represents $z + i\sqrt{3}z$.



i) Explain why $OPRQ$ is a rectangle. **2**

ii) Show that $|z + i\sqrt{3}z| = 2$ **1**

iii) Show that $\arg(z + i\sqrt{3}z) = \alpha + \frac{\pi}{3}$ **1**

iv) By considering the imaginary part of $z + i\sqrt{3}z$, deduce that **1**

$$\sin\alpha + \sqrt{3}\cos\alpha = 2\sin\left(\alpha + \frac{\pi}{3}\right)$$

End of Questions 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) A particle moving in a straight line has an acceleration given by $\ddot{x} = x^2 - 5$ where its displacement is x metres from the origin. Initially the particle is at the origin and has velocity $5\ ms^{-1}$. Find the velocity of this particle when it is 3 metres to the right of the origin. **3**

b) A particle is moving in a straight line. Initially, the particle is 9 metres to the right of the origin, moving with a velocity of $-8\sqrt{3}\ ms^{-1}$. The displacement x is given by $x = A\cos(2t + \alpha) + 5$, for some constants A and α.

i) Prove that the particle is undergoing simple harmonic motion. **2**

ii) Find the values of A and α where $A > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**

iii) Find the period of this motion. **1**

iv) When does the particle first reach the centre of motion. **1**

v) Find the maximum speed of this particle. **1**

c) Prove that $\log_5 14$ is irrational. **2**

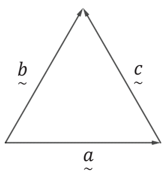
d) Prove using mathematical induction for integers $n \geq 2$, **3**

$$n^{n+1} > n(n+1)^{n-1}$$

End of Questions 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) The sides of this equilateral triangle are 2 units long and represented by the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ as shown. **3**



Evaluate $\underset{\sim}{a} \cdot \left(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}\right)$

b) Use the substitution $t = \tan\dfrac{x}{2}$ to find the value of a, given that **4**

$$\int_0^{\frac{\pi}{2}} \frac{a}{5 + 3\sin x + 4\cos x}\,dx = 1$$

c) i) Determine the equation of the line vector $\underset{\sim}{r}$, given that it passes through the point $(-3, 5, -1)$ and is parallel to the line joining $P(3, 1, -1)$ and $Q(5, 0, 2)$. **1**

ii) Show that the point $(7, 0, 14)$ lies on this line. **1**

d) i) Show that $\left(1 - \sqrt{x}\right)^{n-1}\sqrt{x} = \left(1 - \sqrt{x}\right)^{n-1} - \left(1 - \sqrt{x}\right)^{n}$ **1**

ii) If $I_n = \displaystyle\int_0^1 \left(1 - \sqrt{x}\right)^n dx$ for $n \geq 0$, show that **3**

$I_n = \dfrac{n}{n+2} I_{n-1}$, for $n \geq 1$

iii) Hence evaluate I_4 **2**

End of Questions 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) i) If $p > 0$ and $q > 0$, show that $p + q \geq 2\sqrt{pq}$ **1**

ii) If $p + q = 1$, show that $\dfrac{1}{p} + \dfrac{1}{q} \geq 4$ **2**

b) The tide can be modelled using simple harmonic motion. **4**
The depth of water in a harbour on a particular day is 7.8 metres at low tide and 12.4 metres at high tide. Low tide is at $8{:}15\ am$ and high tide is at $2{:}30\ pm$ on the same day.

If a ship requires a minimum of 9.1 metres to leave the harbour safely. Find between what times the ship can leave safely on that day.

c) Let ω be the complex root of the polynomial $z^7 = 1$ with the smallest possible argument.

i) Explain why $\omega^7 = 1$ and $1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$. **1**

ii) Let $\alpha = \omega + \omega^2 + \omega^4$ and $\beta = \omega^3 + \omega^5 + \omega^6$. **2**
Show that $\alpha + \beta = -1$ and $\alpha\beta = 2$.

iii) Hence write a quadratic equation whose roots are α and β. **1**

iv) Show that $\alpha = -\dfrac{1}{2} + \dfrac{\sqrt{7}}{2}i$ and $\beta = -\dfrac{1}{2} - \dfrac{\sqrt{7}}{2}i$ **1**

v) Write down ω in modulus-argument form, and show that **3**

$\cos\dfrac{4\pi}{7} + \cos\dfrac{2\pi}{7} - \cos\dfrac{\pi}{7} = -\dfrac{1}{2}$ and $\sin\dfrac{4\pi}{7} + \sin\dfrac{2\pi}{7} - \sin\dfrac{\pi}{7} = \dfrac{\sqrt{7}}{2}$

End of Paper

Student Name:____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

2020 Mathematics Extension 2 AT4 Trial Solutions

Section 1

Q1	**A** $i^{2020}=1=i^{100}$	1 Mark
Q2	**A** $z=5e^{\frac{i\pi}{6}}$ $z=5\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ $\bar{z}=5\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$ $(\bar{z})^{-1}=5^{-1}\left(\cos\left(-\frac{\pi}{6}\right)-i\sin\left(-\frac{\pi}{6}\right)\right)$ $(\bar{z})^{-1}=\frac{1}{5}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$	1 Mark
Q3	**C** $e^{i\theta}$ shown is in the second quadrant $e^{-i\theta}$ is reflection along the x-axis, which would be in the 3rd quadrant $ie^{-i\theta}$ is rotating 90° anticlockwise	1 Mark
Q4	**B** A: For all real numbers x, there exist some y such that $xy=10$, this is not true for $x=0$ B: For all real numbers x, there exist a real number y such that $y=x^2$ C: There exist some real number x, such that for all real numbers y, $xy=10$. There is not a single number for x that multiplies with every real numer for y to give a results of 10. D: There exist some real number x, such that for all real numbers y, $y=x^2$. There is not a single number for x that squared gives every real number for y.	1 Mark
Q5	**D** "If you are a strong swimmer, then you are unlikely to drown." The contrapositive is the logically equivalent statement. "If you are likely to drown, then you are not a strong swimmer."	1 Mark
Q6	**B** $I=\int_a^b x^9e^{3x^{10}}dx$ Let $u=x^{10}$ $du=10x^9dx$ $x=b,\quad u=b^{10}$ $x=a,\quad u=a^{10}$ $I=\frac{1}{10}\int_a^b 10x^9e^{3x^{10}}dx$ $I=\frac{1}{10}\int_{a^{10}}^{b^{10}} e^{3u}du$	1 Mark

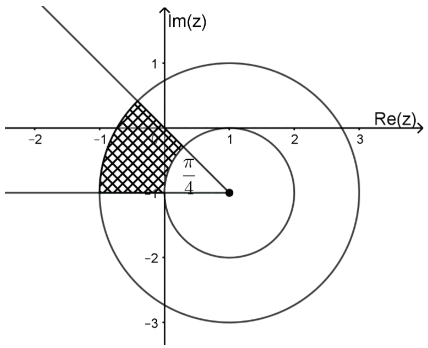
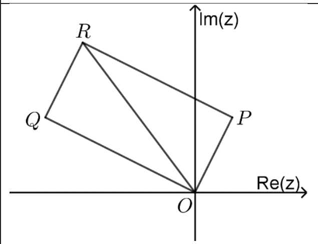
Q7	**D** Vector projection $\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\underset{\sim}{b}$ $=\dfrac{2\times 3+(-5)\times(-1)+(10\times -5)}{3^2+(-1)^2+(-5)^2}\left(3\underset{\sim}{i}-\underset{\sim}{j}-5\underset{\sim}{k}\right)$ $=-\dfrac{39}{35}\left(3\underset{\sim}{i}-\underset{\sim}{j}-5\underset{\sim}{k}\right)$	1 Mark
Q8	**C** $\ddot{x}=-n^2(x-c)$ $\ddot{x}=100-25x$ $\ddot{x}=-25(x-4)$	1 Mark
Q9	**C** $v=2\tan^{-1}x$ $\ddot{x}=v\dfrac{dv}{dx}$ $\ddot{x}=2\tan^{-1}x\times\dfrac{2}{1+x^2}$ $\ddot{x}=\dfrac{4\tan^{-1}x}{1+x^2}$	1 Mark
Q10	**B** $x=1-\sqrt{a}\cos t$ $\cos t=\dfrac{1}{\sqrt{a}}(1-x)$ $\cos^2 t=\dfrac{1}{a}(1-x)^2$ $y=1-\dfrac{1}{b}\sin t$ $\sin t=b(1-y)$ $\sin^2 t=b^2(1-y)^2$ $\cos^2 t+\sin^2 t=1$ $\dfrac{1}{a}(1-x)^2+b^2(1-y)^2=1$ This is a circle if $\dfrac{1}{a}=b^2$ $ab^2=1$	1 Mark

Section 2		
Q11ai	z_1-z_2 $=1-2i-(3+4i)$ $=-2-6i$	1 Mark Correct solution
Q11aii	$z_1\times\bar{z}_2$ $=(1-2i)\times(3-4i)$ $=3-4i-6i-8$ $=-5-10i$	1 Mark Correct solution
Q11aiii	$\dfrac{2+i}{z_2}-\dfrac{z_1}{5i}$ $=\dfrac{2+i}{3+4i}\times\dfrac{3-4i}{3-4i}-\dfrac{1-2i}{5i}\times\dfrac{5i}{5i}$ $=\dfrac{6-8i+3i+4}{3^2+4^2}-\dfrac{5i+10}{-5^2}$ $=\dfrac{10-5i}{25}+\dfrac{5i+10}{25}$ $=\dfrac{20}{25}$ $=\dfrac{4}{5}$	2 Marks Correct solution 1 Mark Rationalise the denominators
Q11b	$2z+\bar{z}=21-8i$ $2(x+yi)+x-yi=21-8i$ $3x+yi=21-8i$ Match the real and the imaginary component $3x=21$ $x=7$ $y=-8$	2 Marks Correct solution 1 Mark Expresses $2z+\bar{z}$ in real and imaginary component
Q11ci	Let $z=-\sqrt{3}+3i$ $\lvert z\rvert=\sqrt{\left(\sqrt{3}\right)^2+3^2}$ $\lvert z\rvert=\sqrt{12}$ $\lvert z\rvert=2\sqrt{3}$ $\arg(z)=\tan^{-1}\left(\dfrac{3}{-\sqrt{3}}\right)$ $\arg(z)=\dfrac{2\pi}{3}$ $-\sqrt{3}+3i=2\sqrt{3}\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$	2 Marks Correct solution 1 Mark Finds the correct modulus or argument
Q11cii	$\left(-\sqrt{3}+3i\right)^5=\left[2\sqrt{3}\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)\right]^5$ Using De Moivre's theorem $\left(-\sqrt{3}+3i\right)^5=\left(2\sqrt{3}\right)^5\left(\cos\dfrac{10\pi}{3}+i\sin\dfrac{10\pi}{3}\right)$ $\left(-\sqrt{3}+3i\right)^5=288\sqrt{3}\left(-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)$ $\left(-\sqrt{3}+3i\right)^5=-144\sqrt{3}-432i$	2 Marks Correct solution 1 Mark Applies De Moivre's theorem correctly

Q11di	$(x+iy)^2 = 9-40i$ $x^2+2xyi-y^2 = 9-40i$ Match up the real and imaginary component $x^2-y^2=9 \quad (1)$ $2xy = 40$ $y = -\frac{20}{x} \qquad (2)$ Sub (2) into (1) $x^2 - \left(\frac{20}{x}\right)^2 = 9$ $x^4 - 400 = 9x^2$ $x^4 - 9x^2 - 400 = 0$ $(x^2-25)(x^2+16) = 0$ $x = \pm 5$(since x is real) $y = \mp 4$ $x+iy = \pm(5-4i)$	2 Marks Correct solution 1 Mark Finds the correct values of x or y
Q11dii	$z^2-7z+10+10i=0$ $z = \frac{7\pm\sqrt{7^2-4\times 1\times(10+10i)}}{2\times 1}$ $z = \frac{7\pm\sqrt{9-40i}}{2\times 1}$ $z = \frac{7\pm(5-4i)}{2}$ $z = \frac{7+5-4i}{2}$ or $\frac{7-5+4i}{2}$ $z = \frac{12-4i}{2}$ or $\frac{2+4i}{2}$ $z = 6-2i$ or $1+2i$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Applies quadratic formula correctly
Q12ai	$\int \frac{2dx}{x^2-8x+21}$ $= 2\int \frac{dx}{(x^2-8x+16)+5}$ $= 2\int \frac{dx}{(x-4)^2+5}$ $= 2\times\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-4}{\sqrt{5}}\right)+c$ $= \frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{x-4}{\sqrt{5}}\right)+c$	2 Marks Correct solution 1 Mark Makes significant progress
Q12aii	$I = \int x^2\log_e x\,dx$ $u = \log_e x \qquad v' = x^2$ $u' = \frac{1}{x} \qquad v = \frac{x^3}{3}$ $I = \frac{x^3}{3}\log_e x - \int \frac{1}{x}\times\frac{x^3}{3}dx$ $I = \frac{x^3}{3}\log_e x - \frac{1}{3}\int x^2 dx$ $I = \frac{x^3}{3}\log_e x - \frac{1}{3}\times\frac{x^3}{3}+c$ $I = \frac{x^3}{3}\log_e x - \frac{x^3}{9}+c$	2 Marks Correct solution 1 Mark Applies integration by parts correctly

Q12b	$I = \int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ Let $u = e^{2x}+e^{-2x}$ $du = (2e^{2x}-2e^{-2x})dx$ $I = \frac{1}{2}\int\frac{2e^{2x}-2e^{-2x}}{e^{2x}+e^{-2x}}dx$ $I = \frac{1}{2}\int\frac{1}{u}du$ $I = \frac{1}{2}\ln\lvert u\rvert + c$ $I = \frac{1}{2}\ln\lvert e^{2x}+e^{-2x}\rvert + c$ $I = \frac{1}{2}\ln\left\lvert\frac{e^{4x}+1}{e^{2x}}\right\rvert + c$ $I = \frac{1}{2}(\ln(e^{4x}+1)-\ln(e^{2x}))+c$ $I = \frac{1}{2}(\ln(e^{4x}+1)-2x\ln(e))+c$ $I = \frac{1}{2}(\ln(e^{4x}+1)-2x)+c$ $I = \ln\sqrt{e^{4x}+1}-x+c$	3 Marks Correct solution 2 Marks Correct primitive 1 Mark Correct integrand in terms of u
Q12ci	$\frac{6x^2+7x-3}{(x-5)(x^2+1)} = \frac{A}{x-5}+\frac{Bx+C}{x^2+1}$ $A(x^2+1)+(Bx+C)(x-5) = 6x^2+7x-3$ Let $x=5$ $A(5^2+1)+(B\times 5+C)\times(5-5) = 6\times 5^2+7\times 5-3$ $26A = 182$ $A = 7$ Let $x = 0$ $7\times(0^2+1)+(B\times 0+C)\times(0-5) = 6\times 0^2+7\times 0-3$ $7-5C = -3$ $-5C = -10$ $C = 2$ Let $x = 1$ $7\times(1^2+1)+(B\times 1+2)\times(1-5) = 6\times 1^2+7\times 1-3$ $14-4B-8 = 10$ $-4B = 4$ $B = -1$	2 Marks Correct solution 1 Mark Finds the correct value for A or B or C
Q12cii	$\int\frac{6x^2+7x-3}{(x-5)(x^2+1)}dx$ $= \int\left(\frac{7}{x-5}+\frac{-x+2}{x^2+1}\right)dx$ $= \int\frac{7}{x-5}dx+\int\frac{-x}{x^2+1}dx+\int\frac{2}{x^2+1}dx$ $= 7\ln\lvert x-5\rvert - \frac{1}{2}\ln(x^2+1)+2\tan^{-1}x+c$	2 Marks Correct solution 1 Mark Makes significant progress
Q12di	$\sin(A+B)+\sin(A-B) = 2\sin A\cos B$ $LHS = \sin(A+B)+\sin(A-B)$ $LHS = \sin A\cos B+\sin B\cos A+\sin A\cos B-\sin B\cos A$ $LHS = 2\sin A\cos B$	1 Mark Correct solution

Q12dii	$\int_0^{\frac{\pi}{2}} \sin 9x \cos 6x \, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin 9x \cos 6x \, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin(9x+6x) + \sin(9x-6x))dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 15x + \sin 3x)dx$ $= \frac{1}{2}\left[-\frac{1}{15}\cos 15x - \frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{2}}$ $= \frac{1}{6}\left[\left(-\frac{1}{5}\cos\left(15\times\frac{\pi}{2}\right) - \cos\left(3\times\frac{\pi}{2}\right)\right) - \left(-\frac{1}{5}\cos(0) - \cos(0)\right)\right]$ $= \frac{1}{6}\left(0 - \left(-\frac{6}{5}\right)\right)$ $= \frac{1}{5}$	3 Marks Correct solution 2 Marks Correct primitive function 1 Mark Rewrite $\sin 9x \cos 6x$ in terms of $\sin 15x + \sin 3x$
Q13ai	$\overrightarrow{AB} = \left(-\underset{\sim}{j} + 4\underset{\sim}{k}\right) - \left(4\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}\right)$ $\overrightarrow{AB} = -4\underset{\sim}{i} + \underset{\sim}{j} + 7\underset{\sim}{k}$	1 Mark Correct solution
Q13aii	$\overrightarrow{BC} = \left(-4\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}\right) - \left(-\underset{\sim}{j} + 4\underset{\sim}{k}\right)$ $\overrightarrow{BC} = -4\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ $\overrightarrow{AB}\cdot\overrightarrow{BC} = \left(-4\underset{\sim}{i} + \underset{\sim}{j} + 7\underset{\sim}{k}\right)\cdot\left(-4\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}\right)$ $\overrightarrow{AB}\cdot\overrightarrow{BC} = (-4)\times(-4) + 1\times 5 + 7\times(-3)$ $\overrightarrow{AB}\cdot\overrightarrow{BC} = 0$ $AB \perp BC$ $\therefore$ The triangle has a right angle at B	2 Marks Correct solution 1 Mark Finds the correct vector for $\overrightarrow{BC}$
Q13aiii	Hypotenuse is AC $\overrightarrow{AC} = \left(-4\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}\right) - \left(4\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}\right)$ $\overrightarrow{AC} = -8\underset{\sim}{i} + 6\underset{\sim}{j} + 4\underset{\sim}{k}$ $\lvert\overrightarrow{AC}\rvert = \sqrt{(-8)^2 + 6^2 + 4^2}$ $\lvert\overrightarrow{AC}\rvert = \sqrt{116}$ $\lvert\overrightarrow{AC}\rvert = 2\sqrt{29}$	1 Mark Correct solution
Q13bi	$\overrightarrow{LM} = \left(7\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}\right) - \left(-5\underset{\sim}{i} + 6\underset{\sim}{j} - 5\underset{\sim}{k}\right)$ $\overrightarrow{LM} = 12\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}$ $\overrightarrow{LM} = 4\left(3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$ $\overrightarrow{MN} = \left(10\underset{\sim}{i} - 4\underset{\sim}{j}\right) - \left(7\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}\right)$ $\overrightarrow{MN} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ $\overrightarrow{LM} = 4\overrightarrow{MN}$ $\overrightarrow{LM}$ and $\overrightarrow{MN}$ are parallel and have M as a common point $\therefore L, M, N$ are collinear	2 Marks Correct solution 1 Mark Finds $\overrightarrow{LM}$ or $\overrightarrow{MN}$
Q13bii	M divides LN in the ratio of $4:1$	1 Mark Correct solution

Q13c	$1 \le \lvert z - 1 + i\rvert \le 2 \cap \frac{3\pi}{4} \le \arg(z - 1 + i) \le \pi$ 	3 Marks Correct solution 2 Mark Makes significant progress 1 Mark Correct annulus or angle shown
Q13di	 $\overrightarrow{OP} + \overrightarrow{OQ} = z + i\sqrt{3}z$ $\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR}$ $\overrightarrow{OP} = \overrightarrow{QR}$ $\overrightarrow{OQ} = \overrightarrow{PR}$ $\therefore OPRQ$ is a parallelogram $\overrightarrow{OQ} = i\sqrt{3}z$ $\overrightarrow{OQ} = i\sqrt{3}\times\overrightarrow{OP}$ Multiplying by i corresponds to a rotation of 90° in an anticlockwise direction. $\therefore \angle POQ = 90°$ $\therefore OPRQ$ is a rectangle	2 Marks Correct solution 1 Mark Either shows $OPRQ$ is a parallelogram or shows $OQ \perp OP$
Q13dii	$\lvert z + i\sqrt{3}z\rvert = \sqrt{OP^2 + RP^2}$ (Pythagoras' theorem) $OP = \lvert z\rvert = 1$ $RP = OQ = \lvert i\sqrt{3}z\rvert = \lvert i\rvert\lvert\sqrt{3}\rvert\lvert z\rvert = \sqrt{3}$ $OR = \lvert z + i\sqrt{3}z\rvert = \sqrt{1+3} = 2$	1 Mark Correct solution
Q13diii	$\arg(z + i\sqrt{3}z) = \arg[z(1 + i\sqrt{3})]$ $\arg(z + i\sqrt{3}z) = \arg(z) + \arg(1 + i\sqrt{3})$ $\arg(z + i\sqrt{3}z) = \alpha + \frac{\pi}{3}$	1 Mark Correct solution

Q13div	$Im(z+i\sqrt{3}z)=Im\left[2\left(\cos\left(\alpha+\frac{\pi}{3}\right)+i\sin\left(\alpha+\frac{\pi}{3}\right)\right)\right]$ $Im(z+i\sqrt{3}z)=2\sin\left(\alpha+\frac{\pi}{3}\right)\quad(1)$ Also $Im(z+i\sqrt{3}z)=Im(z)+Im(i\sqrt{3}z)$ $Im(z+i\sqrt{3}z)=\sin\alpha+Im(i\sqrt{3}\cos\alpha-\sqrt{3}\sin\alpha)$ $Im(z+i\sqrt{3}z)=\sin\alpha+\sqrt{3}\cos\alpha\quad(2)$ $\sin\alpha+\sqrt{3}\cos\alpha=2\sin\left(\alpha+\frac{\pi}{3}\right)$	1 Mark Correct solution
Q14a	$\ddot{x}=x^2-5$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=x^2-5$ $\frac{1}{2}v^2=\frac{x^3}{3}-5x+c$ When $t=0, v=5, x=0$ $\frac{1}{2}\times5^2=0-0+c$ $c=\frac{25}{2}$ $\frac{1}{2}v^2=\frac{x^3}{3}-5x+\frac{25}{2}$ $v^2=\frac{2x^3}{3}-10x+25$ $v=\pm\sqrt{\frac{2x^3}{3}-10x+25}$ The condition $v=5$ when $x=0$ is satisfied by $v=\sqrt{\frac{2x^3}{3}-10x+25}$ When $x=3$ $v=\sqrt{\frac{2\times3^3}{3}-10\times3+25}$ $v=\sqrt{13}\ m/s$	3 Marks Correct solution 2 Marks Determines the correct v 1 Mark Finds v^2 in terms of x
Q14bi	$x=A\cos(2t+\alpha)+5$ $\dot{x}=-2A\sin(2t+\alpha)$ $\ddot{x}=-4A\cos(2t+\alpha)$ $\ddot{x}=-4(x-5)$ $\ddot{x}=-2^2(x-5)$ This is in the form $\ddot{x}=-n^2(x-c)$, where $n=2, c=5$ ∴ This is simple harmonic motion.	2 Marks Correct solution 1 Mark Correct differentiation of $\dot{x}$ and $\ddot{x}$
Q14bii	When $t=0, x=9$ $A\cos\alpha+5=9$ $A\cos\alpha=4\quad(1)$	2 Marks Correct solution
	When $t=0, x=-8\sqrt{3}$ $-2A\sin\alpha=-8\sqrt{3}$ $A\sin\alpha=4\sqrt{3}\quad(2)$ $(1)^2+(2)^2$ $A^2\cos^2\alpha+A^2\sin^2\alpha=64$ $A=8$ $(2)\div(1)$ $\frac{A\sin\alpha}{A\cos\alpha}=\frac{4\sqrt{3}}{4}$ $\tan\alpha=\sqrt{3}$ $\alpha=\frac{\pi}{3}$	1 Mark Finds the value of A or α
Q14biii	$T=\frac{2\pi}{n}$ $T=\pi$	1 Mark Correct solution
Q14biv	Particle is at the centre of motion when $x=5$ $8\cos\left(2t+\frac{\pi}{3}\right)+5=5$ $\cos\left(2t+\frac{\pi}{3}\right)=0$ $2t+\frac{\pi}{3}=\frac{\pi}{2}$ $t=\frac{\pi}{12}$ The particle first reach the centre of motion at $\frac{\pi}{12}$ seconds	1 Mark Correct solution
Q14bv	$\dot{x}=-16\sin\left(2t+\frac{\pi}{3}\right)$ Max speed is $16\ m/s$	1 Mark Correct solution
Q14c	Proof by contradiction: Assume that $\log_5 14$ is rational $\log_5 14=\frac{p}{q}$ Where $p, q\in Z$ with no common factor except 1 $q\log_5 14=p$ $\log_5 14^q=p$ $14^q=5^p$ LHS is even, RHS is odd. This is not possible. ∴ $\log_5 14$ is irrational	2 Marks Correct solution 1 Mark Attempts proof by contradiction
Q14d	RTP: $n^{n+1}>n(n+1)^{n-1}$ for $n\geq2$ 1. Prove statement is true for $n=2$ $LHS=2^{2+1}\qquad RHS=2\times(2+1)^{2-1}$ $LHS=8\qquad RHS=6$ $LHS>RHS$ ∴ Statement is true for $n=2$	3 Marks Correct solution 2 Marks Makes significant progress

	2. Assume statement is true for $n = k$ (k some positive integer ≥ 2) $k^{k+1} > k(k+1)^{k-1}$ $k^k > (k+1)^{k-1}$ (since k is positive) $\dfrac{k^k}{(k+1)^{k-1}} > 1$ 3. Prove statement is true for $n = k+1$ i.e. $(k+1)^{k+1+1} > (k+1)(k+1+1)^{k-1+1}$ $(k+1)^{k+2} > (k+1)(k+2)^k$ $(k+1)^{k+1} > (k+2)^k$ $\dfrac{(k+1)^{k+1}}{(k+2)^k} > 1$ $LHS = \dfrac{(k+1)^{k+1}}{(k+2)^k}$ $LHS > \dfrac{(k+1)^{k+1}}{(k+2)^k} \times \dfrac{(k+1)^{k-1}}{k^k}$ From assumption $\left[\dfrac{k^k}{(k+1)^{k-1}} > 1 \rightarrow \dfrac{(k+1)^{k-1}}{k^k} < 1\right]$ $LHS > \dfrac{(k+1)^{k+1+k-1}}{[k(k+2)]^k}$ $LHS > \dfrac{(k+1)^{2k}}{[k(k+2)]^k}$ $LHS > \dfrac{((k+1)^2)^k}{[k(k+2)]^k}$ $LHS > \left(\dfrac{k^2+2k+1}{k^2+2k}\right)^k$ $LHS > \left(\dfrac{k^2+2k}{k^2+2k} + \dfrac{1}{k^2+2k}\right)^k$ $LHS > \left(1 + \dfrac{1}{k^2+2k}\right)^k$ $LHS > 1$ $\therefore \dfrac{(k+1)^{k+1}}{(k+2)^k} > 1$ $\therefore$ This is true by mathematical induction for all positive integers $n \geq 2$	1 Mark Proves initial case
Q15a	$\underset{\sim}{a} \cdot \left(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}\right)$ $= \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c}$ $= \left\|\underset{\sim}{a}\right\|^2 + \left\|\underset{\sim}{a}\right\| \left\|\underset{\sim}{b}\right\| \cos\dfrac{\pi}{3} + \left\|\underset{\sim}{a}\right\| \left\|\underset{\sim}{c}\right\| \cos\dfrac{2\pi}{3}$ $= 2^2 + 2 \times 2 \times \dfrac{1}{2} + 2 \times 2 \times -\dfrac{1}{2}$ $= 4$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Applies distributive properties

Q15b	$t = \tan\dfrac{x}{2}$ $dt = \dfrac{1}{2}\sec^2\dfrac{x}{2}dx$ $dt = \dfrac{1}{2}(1+t^2)dx$ $dx = \dfrac{2}{1+t^2}dt$ $x = \dfrac{\pi}{2} \quad t = 1$ $x = 0 \quad t = 0$ $\displaystyle\int_0^{\frac{\pi}{2}} \frac{a}{5 + 3\sin x + 4\cos x}dx = 1$ $\displaystyle a\int_0^1 \frac{1}{5 + 3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}dt = 1$ $\displaystyle 2a\int_0^1 \frac{1}{5(1+t^2) + 6t + 4(1-t^2)}dt = 1$ $\displaystyle 2a\int_0^1 \frac{1}{5 + 5t^2 + 6t + 4 - 4t^2}dt = 1$ $\displaystyle 2a\int_0^1 \frac{1}{t^2 + 6t + 9}dt = 1$ $\displaystyle 2a\int_0^1 \frac{1}{(t+3)^2}dt = 1$ $2a\left[\dfrac{(t+3)^{-1}}{-1}\right]_0^1 = 1$ $-2a\left(\dfrac{1}{1+3} - \dfrac{1}{0+3}\right) = 1$ $-2a \times -\dfrac{1}{12} = 1$ $a = 6$	4 Marks Correct solution 3 Marks Makes significant progress 2 Marks Finds the primitive function 1 Mark Shows the integrand in terms of t
Q15ci	$\overrightarrow{PQ} = \begin{pmatrix}5\\0\\2\end{pmatrix} - \begin{pmatrix}3\\1\\-1\end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$ $\underset{\sim}{r} = \begin{pmatrix}-3\\5\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$	1 Mark Correct solution
Q15cii	$7 = -3 + 2\lambda$ $\lambda = 5$ $\begin{pmatrix}7\\0\\14\end{pmatrix} = \begin{pmatrix}-3\\5\\-1\end{pmatrix} + 5\begin{pmatrix}2\\-1\\3\end{pmatrix}$	1 Mark Correct solution
Q15di	$RHS = \left(1-\sqrt{x}\right)^{n-1} - \left(1-\sqrt{x}\right)^n$ $RHS = \left(1-\sqrt{x}\right)^{n-1}\left(1 - \left(1-\sqrt{x}\right)\right)$ $RHS = \left(1-\sqrt{x}\right)^{n-1}\left(1 - 1 + \sqrt{x}\right)$ $RHS = \left(1-\sqrt{x}\right)^{n-1}\sqrt{x}$ $RHS = LHS$	1 Mark Correct solution

Q15dii	$I_n = \int_0^1 \left(1-\sqrt{x}\right)^n dx$ $u = \left(1-\sqrt{x}\right)^n \qquad v' = 1$ $u' = -\frac{n}{2\sqrt{x}}\left(1-\sqrt{x}\right)^{n-1} \qquad v = x$ $I_n = \left[x\left(1-\sqrt{x}\right)^n\right]_0^1 - \int_0^1 \left(-\frac{n}{2\sqrt{x}}\left(1-\sqrt{x}\right)^{n-1} \times x\right) dx$ $I_n = \left[1\left(1-\sqrt{1}\right)^n - 0\left(1-\sqrt{0}\right)^n\right]_0^1 + \frac{n}{2}\int_0^1 \left(\sqrt{x}\left(1-\sqrt{x}\right)^{n-1}\right) dx$ $I_n = 0 + \frac{n}{2}\int_0^1 \left(\sqrt{x}\left(1-\sqrt{x}\right)^{n-1}\right) dx$ $I_n = \frac{n}{2}\int_0^1 \left(\left(1-\sqrt{x}\right)^{n-1} - \left(1-\sqrt{x}\right)^n\right) dx$ $I_n = \frac{n}{2}(I_{n-1} - I_n)$ $I_n + \frac{n}{2}I_n = \frac{n}{2}I_{n-1}$ $\left(1+\frac{n}{2}\right)I_n = \frac{n}{2}I_{n-1}$ $I_n = \frac{\frac{n}{2}I_{n-1}}{\left(1+\frac{n}{2}\right)}$ $I_n = \frac{n}{2}I_{n-1} \times \frac{2}{n+2}$ $I_n = \frac{n}{2}I_{n-1} \times \frac{2}{n+2}$ $I_n = \frac{n}{n+2}I_{n-1}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Applies integration by parts correctly
Q15diii	$I_0 = \int_0^1 dx$ $I_0 = [x]_0^1$ $I_0 = 1$ $I_1 = \frac{1}{3}I_0$ $I_1 = \frac{1}{3}$ $I_2 = \frac{2}{4} \times I_1$ $I_2 = \frac{1}{6}$ $I_3 = \frac{3}{5} \times I_2$ $I_3 = \frac{1}{10}$ $I_4 = \frac{4}{6} \times I_3$ $I_4 = \frac{1}{15}$	2 Marks Correct solution 1 Mark Finds the correct value of I_0
Q16ai	$\left(\sqrt{p}+\sqrt{q}\right)^2 \geq 0$ $p - 2\sqrt{pq} + q \geq 0$ $p + q \geq 2\sqrt{pq}$	1 Mark Correct solution
Q16aii	$\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq}$ From part i) $p + q \geq 2\sqrt{pq}$ $\frac{p+q}{2} \geq \sqrt{pq}$ $\frac{(p+q)^2}{4} \geq pq$ $\frac{4}{(p+q)^2} \leq \frac{1}{pq}$ $\frac{4}{(p+q)^2} \times (p+q) \leq \frac{1}{pq} \times (p+q)$ $\frac{4}{p+q} \leq \frac{p+q}{pq}$ $\frac{p+q}{pq} \geq \frac{4}{p+q}$ $\frac{1}{p} + \frac{1}{q} \geq \frac{4}{1}$ $\therefore \frac{1}{p} + \frac{1}{q} \geq 4$	2 Marks Correct solution 1 Mark Makes significant progress
Q16b	Period $T = 2 \times (14{:}30 - 8{:}15) = 12.5$ hours Amplitude $\frac{1}{2}(12.4 - 7.8) = 2.3$ metres Since the motion is simple harmonic $\ddot{x} = -n^2 x$ $T = \frac{2\pi}{n}$ $n = \frac{2\pi}{12.5}$ $n = \frac{4\pi}{25}$ Using when the tide is 10.1 metres (centre of motion), and when $t = 0$, it's low tide $x = 10.1 - 2.3\cos\left(\frac{4\pi}{25}t\right)$ Minimum depth required is 9.1 metres. $9.1 = 10.1 - 2.3\cos\left(\frac{4\pi}{25}t\right)$ $-1 = -2.3\cos\left(\frac{4\pi}{25}t\right)$ $\frac{10}{23} = \cos\left(\frac{4\pi}{25}t\right)$ $\frac{4\pi}{25}t = \cos^{-1}\frac{10}{23}, \quad 2\pi - \cos^{-1}\frac{10}{23}$ $t = \frac{25}{4\pi}\left(\cos^{-1}\frac{10}{23}\right), \quad \frac{25}{4\pi}\left(2\pi - \cos^{-1}\frac{10}{23}\right)$ $t = 2.2301\ldots, \;\; 10.2698\ldots$ $t \approx 2h14min, \;\; 10h16min$ $8{:}15am + 2h14min = 10{:}29am$ $8{:}15am + 10h16min = 18{:}16 = 6{:}31pm$	4 Marks Correct solution 3 Marks Finds the correct t values 2 Marks Finds the correct motion equation 1 Mark Correct period or amplitude

Q16ci	ω is a complex root of $z^7=1$ $\therefore \omega^7=1$ $z^7-1=(z-1)(z^6+z^5+z^4+z^3+z^2+z+1)$ Since ω is a complex root, $\omega\neq 1$. So ω must be a root of $z^6+z^5+z^4+z^3+z^2+z+1=0$ $\therefore 1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6=0$	1 Mark Correct solution
Q16cii	$\alpha=\omega+\omega^2+\omega^4$ and $\beta=\omega^3+\omega^5+\omega^6$ $\alpha+\beta=\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6$ Since $1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6=0$ Then $\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6=-1$ $\alpha+\beta=-1$ $\alpha\beta=(\omega+\omega^2+\omega^4)(\omega^3+\omega^5+\omega^6)$ $\alpha\beta=\omega^4+\omega^6+\omega^7+\omega^5+\omega^7+\omega^8+\omega^7+\omega^9+\omega^{10}$ $\alpha\beta=\omega^4+\omega^6+1+\omega^5+1+\omega+1+\omega^2+\omega^3$ $\alpha\beta=1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6+2$ $\alpha\beta=2$	2 Marks Correct solution 1 Mark Shows either the correct sum or the product
Q16ciii	$x^2+\frac{b}{a}x+\frac{c}{a}=0$ $\alpha+\beta=-\frac{b}{a}$ $\alpha\beta=\frac{c}{a}$ $x^2+x+2=0$	1 Mark Correct solution
Q16civ	$x=\frac{-1\pm\sqrt{1^2-4\times1\times2}}{2\times1}$ $x=\frac{-1\pm\sqrt{-7}}{2}$ $x=\frac{-1\pm\sqrt{7}i}{2}$ $\therefore \alpha=-\frac{1}{2}+\frac{\sqrt{7}}{2}i, \beta=-\frac{1}{2}-\frac{\sqrt{7}}{2}i$	1 Mark Correct solution
Q16cv	$z^7=1$ $z=\left(\cos\frac{2\pi k}{7}+i\sin\frac{2\pi k}{7}\right), k=0,1,2,3,4,5,6$ Since ω has the smallest possible argument, $\omega=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$ $\alpha=\omega+\omega^2+\omega^4$ $\alpha=\left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)+\left(\cos\frac{4\pi}{7}+i\sin\frac{4\pi}{7}\right)+\left(\cos\frac{8\pi}{7}+i\sin\frac{8\pi}{7}\right)$ $\alpha=-\frac{1}{2}+\frac{\sqrt{7}}{2}i$ Equating real and imaginary parts $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}=-\frac{1}{2}$ $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}-\cos\frac{\pi}{7}=-\frac{1}{2}$ $\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}=\frac{\sqrt{7}}{2}$ $\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{\pi}{7}=\frac{\sqrt{7}}{2}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Finds ω in mod-arg form